Eur. Phys. J. C 42, 271–279 (2005)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02285-x

THE EUROPEAN
PHYSICAL JOURNAL C

Twist-3 distribution amplitudes of the pion and kaon from the QCD sum rules

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Received: 6 January 2005 / Revised version: 29 March 2005 /
Published online: 22 June 2005 –

Abstract. Twist-3 distribution amplitudes of the pion and kaon are studied in this paper. We calculate the first several moments for the twist-3 distribution amplitudes ($\phi^{\pi}_{p,\sigma}$ and $\phi^{K}_{p,\sigma}$) of the pion and kaon by applying the QCD sum rules. Our results show that (i) the first three moments of ϕ^K_p and the first two moments of ϕ^{π}_p and $\phi^{\pi,K}_{\sigma}$ of the pion and kaon can be obtained with 30% uncertainty; (ii) the fourth moment of the ϕ^{π}_p and the second moment of the ϕ^K_{σ} can be obtained when the uncertainty are relaxed to 35%; (iii) the fourth moment of the ϕ^{π}_{σ} can be obtained only when the uncertainty are relaxed to 40%; (iv) we have $m^p_{0\pi} = 1.10 \pm 0.08\,\mathrm{GeV}$ and $m^p_{0K} = 1.25 \pm 0.15\,\mathrm{GeV}$ after including the α_s-corrections to the perturbative part. These moments will be helpful for constructing the twist-3 wave functions of the pion and kaon.

PACS. 13.20.He, 11.55.Hx

1 Introduction

Hadronic distribution amplitudes, which involve non-perturbative information, are the important ingredients when applying QCD to hard exclusive processes via the factorization theorem. These distribution amplitudes are process-independent and should be determined by the hadronic dynamics. They satisfy the renormalization group equation and have asymptotic solutions as $Q^2 \to \infty$.

From the counting rule, the twist-2 distribution amplitude makes the leading contribution and the contribution from the higher-twist distribution amplitude is suppressed by a factor $1/Q^2$ in the large momentum transfer regions. However as one wants to explain the present experimental data, the non-leading contributions should be taken into account. The non-leading contributions include higher-order corrections, higher-twist and higher Fock state contributions et cetera. Therefore one has to study the twist-2 and higher-twist distribution amplitudes as universal non-perturbative inputs for the exclusive processes.

Distribution amplitudes can be obtained from the hadronic wave functions by integrating the transverse momenta of the quarks in the hadrons. For example, the pionic distribution amplitudes of the lowest Fock state are defined by

$$\begin{aligned}&\left\langle 0\left|\bar{d}_{\alpha}(z)\,[z,-z]\,u_{\beta}(-z)\right|\pi(q)\right\rangle \\ &= -\frac{\mathrm{i}}{8}f_{\pi}\int_{-1}^{1}\mathrm{d}\xi\;\mathrm{e}^{\mathrm{i}\xi(z\cdot q)} \\ &\times\Big\{\not{q}\gamma_5\phi_{\pi}(\xi) + m^p_{0\pi}\gamma_5\phi^{\pi}_p(\xi) \\ &\quad + \frac{2}{3}m^{\sigma}_{0\pi}\sigma_{\mu\nu}\gamma_5 q^{\mu}z^{\nu}\phi^{\pi}_{\sigma}(\xi)\Big\}_{\beta\alpha} + \dots \end{aligned} \quad (1)$$

where $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu})$, f_{π} is the pion decay constant and

$$[z,-z] = \exp\left\{\mathrm{i}g\int_{-z}^{z}\mathrm{d}x^{\mu}A_{\mu}\right\}$$

is the Wilson line inserted to preserve gauge invariance of the distribution amplitudes. The $\phi_{\pi}(\xi)$, $\phi^{\pi}_p(\xi)$ and $\phi^{\pi}_{\sigma}(\xi)$ in (1) are the twist-2 and two twist-3 (non-leading) distribution amplitudes respectively. For the K meson, the definition is similar except for the d quark being replaced by the s quark and $m^{p,\sigma}_{0\pi}$ replaced by $m^{p,\sigma}_{0K}$.

To isolate the light-cone twist-3 distribution amplitudes ϕ^{π}_p and ϕ^{π}_{σ} of the pion, one can contract (1) with the gamma matrices γ_5 and $\sigma_{\mu\nu}\gamma_5$ respectively,

$$\begin{aligned}&\left\langle 0\left|\bar{d}(z)\mathrm{i}\gamma_5\,[z,-z]\,u(-z)\right|\pi^{+}(q)\right\rangle \\ &= m^p_{0\pi}f_{\pi}\;\frac{1}{2}\int_{-1}^{1}\mathrm{d}\xi\;\phi^{\pi}_p(\xi)\mathrm{e}^{\mathrm{i}\xi(z\cdot q)} + \cdots\end{aligned} \quad (2)$$

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and

$$\left\langle 0\left|\bar{d}(z)\sigma_{\mu\nu}\gamma_5\,[z,-z]\,u(-z)\right|\pi^+(q)\right\rangle = \frac{-\mathrm{i}\;m_{0\pi}^{\sigma}f_\pi}{3}(q_\mu z_\nu - q_\nu z_\mu)\;\frac{1}{2}\int_{-1}^{1}\mathrm{d}\xi\;\phi_\sigma^\pi(\xi)\mathrm{e}^{\mathrm{i}\xi(z\cdot q)} + \cdots \quad (3)$$

In a similar way, we can define two twist-3 distribution amplitudes ϕ_p^K and ϕ_σ^K of the kaon in the following:

$$\left\langle 0\left|\bar{s}(z)\mathrm{i}\gamma_5\,[z,-z]\,u(-z)\right|K^+(q)\right\rangle = m_{0K}^{p}f_K\;\frac{1}{2}\int_{-1}^{1}\mathrm{d}\zeta\;\phi_p^K(\zeta)\mathrm{e}^{\mathrm{i}\zeta(z\cdot q)} + \cdots \quad (4)$$

and

$$\left\langle 0\left|\bar{s}(z)\sigma_{\mu\nu}\gamma_5\,[z,-z]\,u(-z)\right|K^+(q)\right\rangle = \frac{-\mathrm{i}\;m_{0K}^{\sigma}f_K}{3}(q_\mu z_\nu - q_\nu z_\mu)\;\frac{1}{2}\int_{-1}^{1}\mathrm{d}\zeta\;\phi_\sigma^K(\zeta)\mathrm{e}^{\mathrm{i}\zeta(z\cdot q)} + \cdots \quad (5)$$

The dots in the above definitions refer to those higher-twist distribution amplitudes. We do not consider their influences in the following calculation.

In their pioneering work in [1,2], the authors pointed out that the first several moments of the distribution amplitudes could be calculated in the QCD sum rules [3]. Those moments are helpful to construct a model for the hadronic distribution amplitudes.

The parameters $m_{0\pi}^{p,\sigma}$ and $m_{0K}^{p,\sigma}$ introduced in the definition are used to normalize the zeroth moments of their corresponding distribution amplitudes. It is shown in this paper that these parameters determined by the QCD sum rules are smaller than those required by the equations of motion (e.g., see [4,5]).

In this paper, we calculate the first three moments of the twist-3 distribution amplitudes of π and K, defined in (2)–(5), in the QCD sum rules. For the pion case, we had calculated the moments of distribution amplitude ϕ_p^π in a previous paper [6]. However there were some mistakes in estimating the contribution from the continuous spectrum and the Borel windows which would severely influence the values of the moments. Now we present the correct expressions for the moments of ϕ_p^π and re-analyze their numerical results in this paper. Furthermore, it is well known that axial currents in a correlator would couple to instantons (see, for example, [7]). And this may cause some complications in the calculation and make the results unreliable. We will not explore their influences in this paper.

This paper is organized as follows. In Sect. 2, we give the sum rules of the moments of ϕ_p^π and ϕ_σ^π for the π meson. The sum rules for the moments of ϕ_p^K and ϕ_σ^K of K meson are given in Sect. 3. The $SU(3)$ symmetry violation has been taken into account. In Sect. 4, numerical analysis of various moments is presented. The information of the 3-particle twist-3 distribution amplitude obtained from the 2-particle distribution amplitudes are also discussed. The last section is reserved for a summary and discussion.

2 QCD sum rules for the moments of ϕ_p^π and ϕ_σ^π of the pion

In this section we apply the background field method in QCD to calculate the moments [8–13]. Expanding (2) and (3) around $z^2=0$, we have

$$\left\langle 0\left|\bar{d}(0)\gamma_5\left(\mathrm{i}z\cdot\overleftrightarrow{D}\right)^n u(0)\right|\pi^+(q)\right\rangle = -\mathrm{i}f_\pi m_{0\pi}^{p}\left\langle\xi_p^n\right\rangle(z\cdot q)^n \quad (6)$$

and

$$\left\langle 0\left|\bar{d}(0)\sigma_{\mu\nu}\gamma_5\left(\mathrm{i}z\cdot\overleftrightarrow{D}\right)^{n+1} u(0)\right|\pi^+(q)\right\rangle = -\frac{n+1}{3}f_\pi m_{0\pi}^{\sigma}\left\langle\xi_\sigma^n\right\rangle(q_\mu z_\nu - q_\nu z_\mu)(z\cdot q)^n \quad (7)$$

respectively. The moments in (6) and (7) are defined by the following expressions:

$$\left\langle\xi_p^n\right\rangle = \frac{1}{2}\int_{-1}^{1}\xi^n\phi_p^\pi(\xi)\mathrm{d}\xi,\quad \left\langle\xi_\sigma^n\right\rangle = \frac{1}{2}\int_{-1}^{1}\xi^n\phi_\sigma^\pi(\xi)\mathrm{d}\xi. \quad (8)$$

As usual, the $SU(2)$ isospin symmetry can be taken as (nearly) exact. It means that the distribution of longitudinal momentum carried by the quarks (in the light-cone framework) should be symmetric between u and d, i.e., odd moments of the distribution amplitudes $\phi_p^\pi,\phi_\sigma^\pi$ should be zero. So we consider only the even moments for the pion case in the following.

In order to obtain the sum rules of the moments, we introduce two corresponding correlation functions,

$$(z\cdot q)^{2n}\;I_p^{(2n,0)}(q^2) \equiv -\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}q\cdot x} \times\left\langle 0\left|T\left\{\bar{d}(x)\gamma_5(\mathrm{i}z\cdot\overleftrightarrow{D})^{2n}u(x),\bar{u}(0)\gamma_5 d(0)\right\}\right|0\right\rangle \quad (9)$$

and

$$-\mathrm{i}(q_\mu z_\nu - q_\nu z_\mu)(z\cdot q)^{2n}\;I_\sigma^{(2n,0)}(q^2) \equiv -\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}q\cdot x}\left\langle 0\left|T\left\{\bar{d}(x)\sigma_{\mu\nu}\gamma_5(\mathrm{i}z\cdot\overleftrightarrow{D})^{2n+1}u(x),\right.\right.\right. \left.\bar{u}(0)\gamma_5 d(0)\right\}\left|0\right\rangle. \quad (10)$$

In the deep Euclidean region ($-q^2\gg 0$), one can calculate the Wilson coefficients in the operator product expansion (OPE) for (9) and (10) perturbatively. The results with power correction to dimension six and the α_s-corrections to lowest order are written as

$$I_p^{(2n,0)}(q^2)_{\mathrm{QCD}} = -\frac{1}{2n+1}\;\frac{3}{8\pi^2}q^2\ln\frac{-q^2}{\mu^2} - \frac{1}{8}\frac{\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle}{q^2} - \frac{2n-1}{2}\frac{(m_u+m_d)\left\langle\bar{q}q\right\rangle}{q^2}$$

$$+\frac{16\pi}{81}(16n^2+4n+21)\frac{\langle\sqrt{\alpha_s}\bar{q}q\rangle^2}{q^4} \quad (11)$$

and

$$I_\sigma^{(2n,0)}(q^2)_{\rm QCD} = -\frac{1}{2n+3}\frac{3}{8\pi^2}q^2\ln\frac{-q^2}{\mu^2} - \frac{1}{24}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{q^2} - \frac{2n+1}{2}\frac{(m_u+m_d)\langle\bar{q}q\rangle}{q^2} + \frac{16\pi}{81}(16n^2+12n-7)\frac{\langle\sqrt{\alpha_s}\bar{q}q\rangle^2}{q^4}. \quad (12)$$

On the other hand, in the physical region, the correlation functions (9) and (10) can be written in terms of their hadronic spectrum representation (according to (6) and (11), and (7) and (12) respectively),

$$\mathrm{Im}\ I_p^{(2n,0)}(q^2)_{\rm had} = \pi\delta(q^2-m_\pi^2)f_\pi^2(m_{0\pi}^p)^2\langle\xi_p^{2n}\rangle + \pi\frac{3}{8\pi^2}\frac{1}{2n+1}q^2\theta(q^2-s_\pi^p) \quad (13)$$

and

$$\mathrm{Im}\ I_\sigma^{(2n,0)}(q^2)_{\rm had} = \pi\delta(q^2-m_\pi^2)\frac{2n+1}{3}f_\pi^2 m_{0\pi}^\sigma m_{0\pi}^p\langle\xi_\sigma^{2n}\rangle + \pi\frac{3}{8\pi^2}\frac{1}{2n+3}q^2\theta(q^2-s_\pi^\sigma)\ . \quad (14)$$

The correlation function in these two regions can be related by the dispersion relation,

$$\frac{1}{\pi}\int \mathrm{d}s\frac{\mathrm{Im}\ I(s)_{\rm had}}{s+Q^2} = I(-Q^2)_{\rm QCD}.$$

In order to improve its convergence, we apply the Borel transformation,

$$\frac{1}{\pi}\frac{1}{M^2}\int \mathrm{d}s\ \mathrm{e}^{-s/M^2}\ \mathrm{Im}\ I(s)_{\rm had} = \hat{L}_M\ I(-Q^2)_{\rm QCD}, \quad (15)$$

where M is the Borel parameter. Substituting (11) and (13) into (15) gives the sum rules for the moments of ϕ_p^π:

$$\langle\xi_p^{2n}\rangle(m_{0\pi}^p)^2 = \frac{\mathrm{e}^{m_\pi^2/M^2}M^4}{f_\pi^2} \times\left\{\frac{1}{(2n+1)}\frac{3}{8\pi^2}\left[1-\left(1+\frac{s_\pi^p}{M^2}\right)\mathrm{e}^{-s_\pi^p/M^2}\right] + \frac{1}{8}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{M^4} + \frac{2n-1}{2}\frac{(m_u+m_d)\langle\bar{q}q\rangle}{M^4} + \frac{16\pi}{81}(16n^2+4n+21)\frac{\langle\sqrt{\alpha_s}\bar{q}q\rangle^2}{M^6}\right\}. \quad (16)$$

Similarly, substituting (12) and (14) into (15) gives the sum rules for the moments of ϕ_σ^π:

$$\langle\xi_\sigma^{2n}\rangle m_{0\pi}^\sigma m_{0\pi}^p = 3\frac{\mathrm{e}^{m_\pi^2/M^2}M^4}{f_\pi^2} \times\left\{\frac{1}{(2n+1)(2n+3)}\frac{3}{8\pi^2}\left[1-\left(1+\frac{s_\pi^\sigma}{M^2}\right)\mathrm{e}^{-s_\pi^\sigma/M^2}\right] + \frac{1}{24}\frac{1}{2n+1}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{M^4} + \frac{1}{2}\frac{(m_u+m_d)\langle\bar{q}q\rangle}{M^4} + \frac{16\pi}{81}\frac{16n^2+12n-7}{2n+1}\frac{\langle\sqrt{\alpha_s}\bar{q}q\rangle^2}{M^6}\right\}, \quad (17)$$

where s_π^p and s_π^σ are the threshold values to be chosen properly, and the zeroth moment has been normalized to unity, i.e., $\langle\xi_p^0\rangle = \langle\xi_\sigma^0\rangle = 1$.

3 QCD sum rules for the moments of ϕ_p^K and ϕ_σ^K of the kaon

For the kaon, we should consider the difference between s quark and u quark (i.e., the violation of the $SU(3)$ flavor symmetry). There is an asymmetry of the distribution of the longitudinal momentum carried by s quark and u quark in the light-cone framework. So the odd moments of the distribution amplitudes for the K meson do not vanish. The violation effects of the $SU(3)$ flavor symmetry for leading-twist distribution amplitudes of K and/or K^* meson were considered in [14]. So in calculating the odd moments, we retain all the corrections to order m_s^2.

Expanding (4) and (5) around $z^2=0$, one obtains

$$\left\langle 0\left|\bar{s}(0)\gamma_5\left(\mathrm{i}z\cdot\overleftrightarrow{D}\right)^n u(0)\right|K^+(q)\right\rangle = -\mathrm{i}f_K m_{0K}^p\langle\zeta_p^n\rangle(z\cdot q)^n \quad (18)$$

and

$$\left\langle 0\left|\bar{s}(0)\sigma_{\mu\nu}\gamma_5(\mathrm{i}z\cdot\overleftrightarrow{D})^{n+1}u(0)\right|K^+(q)\right\rangle = -\frac{n+1}{3}f_K m_{0K}^\sigma\langle\zeta_\sigma^n\rangle(q_\mu z_\nu - q_\nu z_\mu)(z\cdot q)^n \quad (19)$$

respectively, and the moments are defined by

$$\langle\zeta_p^n\rangle = \frac{1}{2}\int_{-1}^1\zeta^n\ \phi_p^K(\zeta)\mathrm{d}\zeta\ ,\qquad \langle\zeta_\sigma^n\rangle = \frac{1}{2}\int_{-1}^1\zeta^n\ \phi_\sigma^K(\zeta)\mathrm{d}\zeta. \quad (20)$$

Similar to the pion case, the correlation functions for calculating the moments of the kaon are defined as

$$(z\cdot q)^n\ I_{Kp}^{(n,0)}(q^2) \equiv -\mathrm{i}\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}q\cdot x} \times\left\langle 0\left|T\left\{\bar{s}(x)\gamma_5(\mathrm{i}z\cdot\overleftrightarrow{D})^n u(x),\bar{u}(0)\gamma_5 s(0)\right\}\right|0\right\rangle \quad (21)$$

and

$$-\mathrm{i}(q_\mu z_\nu - q_\nu z_\mu)(z\cdot q)^n\ I_{K\sigma}^{(n,0)}(q^2) \equiv -\mathrm{i}\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}q\cdot x}\left\langle 0\left|T\left\{\bar{s}(x)\sigma_{\mu\nu}\gamma_5(\mathrm{i}z\cdot\overleftrightarrow{D})^{n+1}u(x),\right.\right.\right.$$

$$\bar{u}(0)\gamma_5 s(0)\}|\,0\rangle\,. \tag{22}$$

As discussed in the previous section, the correlation functions can be calculated perturbatively in the deep Euclidean region, i.e., $Q^2 = -q^2 \gg 0$. Combined with (18) and (19), we assume the hadronic spectrum representations of the above correlations as follows:

$$\begin{aligned}&\mathrm{Im}\; I_{Kp}^{(n,0)}(q^2)_{\rm had}\\ &= \pi\delta(q^2 - m_K^2) f_K^2 (m_{0K}^p)^2 \left\langle \zeta_p^n \right\rangle\\ &+ \pi \frac{3}{8\pi^2}\frac{1}{n+1} q^2 \theta(q^2 - s_K^p)\end{aligned} \tag{23}$$

and

$$\begin{aligned}&\mathrm{Im}\; I_{K\sigma}^{(n,0)}(q^2)_{\rm had}\\ &= \pi\delta(q^2 - m_K^2)\frac{n+1}{3} f_K^2 m_{0K}^\sigma m_{0K}^p \left\langle \zeta_\sigma^n \right\rangle\\ &+ \pi \frac{3}{8\pi^2}\frac{1}{n+3} q^2 \theta(q^2 - s_K^\sigma).\end{aligned} \tag{24}$$

Employing the dispersion relation and Borel transformation as done in the previous section, the sum rules for the moments of ϕ_p^K can be expressed in the following:

$$\begin{aligned}\left\langle \zeta_p^{2n} \right\rangle (m_{0K}^p)^2 &= \frac{\mathrm{e}^{m_K^2/M^2} M^4}{f_K^2}\\ &\times \left\{ \frac{1}{(2n+1)} \frac{3}{8\pi^2}\left[1 - \left(1 + \frac{s_K^p}{M^2}\right)\mathrm{e}^{-s_K^p/M^2}\right]\right.\\ &+ \frac{1}{8}\frac{\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle}{M^4}\\ &+ \frac{[(2n+1)m_s - 2m_u]\left\langle \bar{s}s \right\rangle}{2M^4}\\ &+ \frac{[(2n+1)m_u - 2m_s]\left\langle \bar{u}u \right\rangle}{2M^4}\\ &+ \frac{16\pi}{81}(8n^2+2n-3)\frac{\alpha_s\left[\left\langle \bar{s}s \right\rangle^2 + \left\langle \bar{u}u \right\rangle^2\right]}{M^6}\\ &\left. + \frac{16\pi}{3}\frac{\alpha_s \left\langle \bar{s}s \right\rangle \left\langle \bar{u}u \right\rangle}{M^6}\right\}\end{aligned} \tag{25}$$

and

$$\begin{aligned}\left\langle \zeta_p^{1} \right\rangle (m_{0K}^p)^2 &= \frac{\mathrm{e}^{m_K^2/M^2} M^4}{f_K^2}\\ &\times \left\{ -\frac{3}{8\pi^2}\frac{m_s^2}{M^2}\left(1 - \mathrm{e}^{-s_K^p/M^2}\right)\right.\\ &+ \frac{(m_s - m_u)\left[\left\langle \bar{u}u \right\rangle + \left\langle \bar{s}s \right\rangle\right]}{M^4}\\ &+ \frac{m_s^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle}{4M^6}\\ &\left. + \frac{4\pi}{27}\frac{m_s^2}{M^2}\frac{36\;\alpha_s \left\langle \bar{u}u \right\rangle \left\langle \bar{s}s \right\rangle - 4\;\alpha_s \left\langle \bar{u}u \right\rangle^2}{M^6}\right\}.\end{aligned} \tag{26}$$

For the twist-3 amplitude ϕ_σ^K, we make a similar calculation according to the above procedure and the sum rules for the moments of ϕ_σ^K become

$$\begin{aligned}\left\langle \zeta_\sigma^{2n} \right\rangle m_{0K}^p m_{0K}^\sigma &= 3\frac{\mathrm{e}^{m_K^2/M^2} M^4}{f_K^2}\\ &\times \left\{ \frac{1}{(2n+1)(2n+3)} \frac{3}{8\pi^2}\left[1 - \left(1 + \frac{s_K^\sigma}{M^2}\right)\mathrm{e}^{-s_K^\sigma/M^2}\right]\right.\\ &+ \frac{1}{24(2n+1)}\frac{\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle}{M^4} + \frac{m_s\left\langle \bar{s}s \right\rangle + m_u \left\langle \bar{u}u \right\rangle}{2M^4}\\ &+ \frac{16\pi}{81}(4n+1)\frac{\alpha_s\left[\left\langle \bar{s}s \right\rangle^2 + \left\langle \bar{u}u \right\rangle^2\right]}{M^6}\\ &\left. - \frac{16\pi}{9(2n+1)}\frac{\alpha_s \left\langle \bar{s}s \right\rangle \left\langle \bar{u}u \right\rangle}{M^6}\right\}\end{aligned} \tag{27}$$

and

$$\begin{aligned}\left\langle \zeta_\sigma^{1} \right\rangle m_{0K}^p m_{0K}^\sigma &= \frac{3}{2}\frac{\mathrm{e}^{m_K^2/M^2} M^4}{f_K^2}\\ &\times \left\{ -\frac{1}{4\pi^2}\frac{m_s^2}{M^2}\left(1 - \mathrm{e}^{-s_K^\sigma/M^2}\right) + \frac{m_s\left\langle \bar{s}s \right\rangle - m_u \left\langle \bar{u}u \right\rangle}{M^4}\right.\\ &+ \frac{m_s^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle}{6M^6}\left(\ln\frac{M^2}{\mu^2} + 1 - \gamma_{\rm E}\right) - \frac{1}{3}\frac{m_s g_s \left\langle \bar{s}\sigma G s \right\rangle}{M^6}\\ &\left. + \frac{32\pi}{27}\frac{\alpha_s\left[\left\langle \bar{s}s \right\rangle^2 - \left\langle \bar{u}u \right\rangle^2\right]}{M^6}\right\},\end{aligned} \tag{28}$$

where $\gamma_{\rm E} = 0.577\,216\cdots$ is the Euler constant, s_K^p and s_K^σ in the above equations are the threshold values to be chosen properly, and the zeroth moments have been normalized, $\left\langle \zeta_p^0 \right\rangle = \left\langle \zeta_\sigma^0 \right\rangle = 1$.

4 Numerical analysis

To analyze the sum rules (16), (17) and (25)–(28) numerically, we take the input parameters as usual: $f_K = 0.160\,\mathrm{GeV}$, $f_\pi = 0.133\,\mathrm{GeV}$, $m_s = 0.156\,\mathrm{GeV}$, $m_u = 0.005\,\mathrm{GeV}$, $m_d = 0.008\,\mathrm{GeV}$, $\left\langle \bar{u}u \right\rangle = \left\langle \bar{d}d \right\rangle = -(0.24\,\mathrm{GeV})^3$, $\left\langle \bar{s}s \right\rangle = 0.8 \left\langle \bar{u}u \right\rangle$, $g_s \left\langle \bar{s}\sigma G s \right\rangle = -0.00885\,\mathrm{GeV}^5$, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle = 0.012\,\mathrm{GeV}^4$, $\alpha_s(1\,\mathrm{GeV}) = 0.5$. The renormalization scale $\mu = M$ is assumed in the following analysis.

As to the threshold values $s_{\pi,K}^{p,\sigma}$ in the sum rules, they can be taken to the mass square of the first excited states in the corresponding channel. Although the windows become broader when the $s_{\pi,K}^{p,\sigma}$ are larger, the threshold values cannot exceed the first excited states. In order to get maximum stability of the sum rules, they are taken as the mass square of the first excited states, i.e.,

$$s_\pi^{p,\sigma} = (1.3\,\mathrm{GeV})^2, \quad s_K^{p,\sigma} = (1.46\,\mathrm{GeV})^2,$$

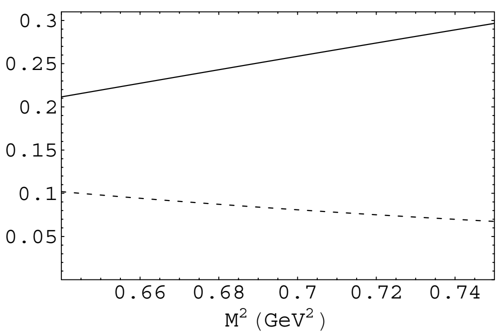
where the first excited state is $\pi'(1300)$ for the pion case, and that for the kaon case is $K(1460)$ [15].

4.1 Determination of the normalization constants

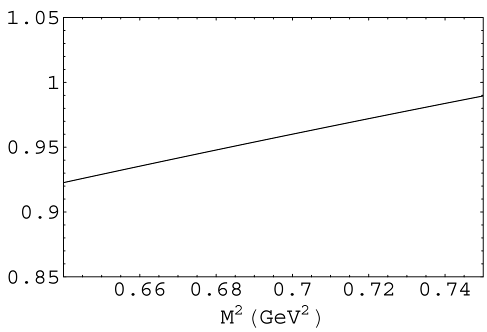
For each distribution amplitude, we introduce a corresponding parameter (e.g., $m_{0\pi}^p$ for ϕ_p^π). These parameters are normalization constants which normalize the zeroth moments to one. Their values can be determined from the sum rules (16), (17), (25) and (27) with $n = 0$. Take $m_{0\pi}^p$ as an example. To identify a Borel window (M^2) for the sum rule of $m_{0\pi}^p$, one requires that the continuum contribution is less than 30% and the dimension-six condensate contribution is less than 10%. This requirement leads to a window $M^2 \in (0.64, 0.75)\,\mathrm{GeV}^2$ and one can find $m_{0\pi}^p = 0.96 \pm 0.03\,\mathrm{GeV}$ within this window. The results are plotted in Figs. 1a,b for $m_{0\pi}^p$.

The same procedure can be applied to get the other parameters $m_{0\pi}^\sigma$, $m_{0K}^{p,\sigma}$, and the numerical results are listed in Table 1. The continuum contributions to the sum rules are required to be less than 30% and the dimension-six contribution is required to be less than 16% for $m_{0\pi}^\sigma$, and 10% for m_{0K}^p and m_{0K}^σ. It should be pointed out that when the α_s-correction to the perturbative part of the sum rule for $m_{0\pi}^p$, m_{0K}^p are taken into account [9], their values will be increased by 15–20%. For example, $m_{0\pi}^p = 1.10 \pm 0.08\,\mathrm{GeV}$ and $m_{0K}^p = 1.25 \pm 0.15\,\mathrm{GeV}$.

One can see from the above that $m_{0\pi}^\sigma$ is smaller than $m_{0\pi}^p$ about 30%. The main reason is the opposite signs of the dimension-six condensate terms in (16) and (17). For the kaon case, the approximate 30% difference between m_{0K}^σ and m_{0K}^p is due to the same reason (see (25) and (27)).



a



b

Fig. 1. a The window for the normalization constant $m_{0\pi}^p$ without α_s-correction in the perturbative part in the sum rule. The dashed line is the ratio of the dimension-six condensate contribution to the total sum rule ($n = 0$) and the solid line is the ratio of the continuum contribution to the total sum rule ($n = 0$). **b** The corresponding values of $m_{0\pi}^p$ within the window

Table 1. The normalization constants m_0 and the corresponding Borel windows for the distribution amplitudes $\phi_{p,\sigma}^\pi$ and $\phi_{p,\sigma}^K$ without α_s-correction in the perturbative parts in the sum rules

	ϕ_p^π	ϕ_σ^π	ϕ_p^K	ϕ_σ^K
m_0 (GeV)	0.96 ± 0.03	0.67 ± 0.06	1.06 ± 0.09	0.71 ± 0.09
M^2 (GeV^2)	0.64–0.75	0.60–0.68	0.58–0.93	0.66–0.83

It was shown that the normalization constants for the twist-3 distribution amplitudes can be obtained from the equations of motion [4]. So at this point, we would like to compare our results with those obtained by the equations of motion and judge upon the accuracy of the sum rules presented above. From the equations of motion follows the normalization constant $m_{0\pi}^p \to \mu_\pi = m_\pi^2/\bar{m}$ for ϕ_p^π and $m_{0\pi}^\sigma \to \widetilde{\mu}_\pi = \mu_\pi - \bar{m}$ for ϕ_σ^π, where $\bar{m} = m_u + m_d$. Furthermore, the Q^2 dependence of the quark mass $\bar{m}$ can be written as $\bar{m}(Q^2) = [\ln(\mu^2/\Lambda^2)/\ln(Q^2/\Lambda^2)]^{4/9}\,\bar{m}(\mu^2)$ which can be obtained from the anomalous dimension of the quark mass. Thus we find $\mu_\pi(1\,\mathrm{GeV}^2) \approx 1.48\,\mathrm{GeV}$ and $\widetilde{\mu}_\pi(1\,\mathrm{GeV}^2) \approx 1.47\,\mathrm{GeV}$ as we take $\bar{m}(4\,\mathrm{GeV}^2) = 11\,\mathrm{MeV}$. For the kaon case, we take $(m_u+m_s)(4\,\mathrm{GeV}^2) = 140\,\mathrm{MeV}$. From the equation of motion we have $\mu_K(1\,\mathrm{GeV}^2) = m_K^2/(m_u + m_s)(1\,\mathrm{GeV}^2) \approx 1.45\,\mathrm{GeV}$ and $(\mu_K - (m_s + m_u))(1\,\mathrm{GeV}^2) \approx 1.28\,\mathrm{GeV}$. In the above statement, the QCD scale $\Lambda = 250\,\mathrm{MeV}$ is assumed and $n_f = 3$ flavors are taken into account. One can see that the deviation of $m_{0\pi}^p$ from μ_π is about 26% and the deviation of m_{0K}^p from μ_K is less than 15% (α_s-corrections to the perturbative parts are included).

If the α_s-correction to the perturbative part is 15–20% and these corrections make the normalization constants increasing, one may expect that the deviation of $m_{0\pi}^\sigma$ from $\widetilde{\mu}_\pi$ is about 45% and the deviation of m_{0K}^σ from $\mu_K - (m_u + m_s)$ is about 33%.

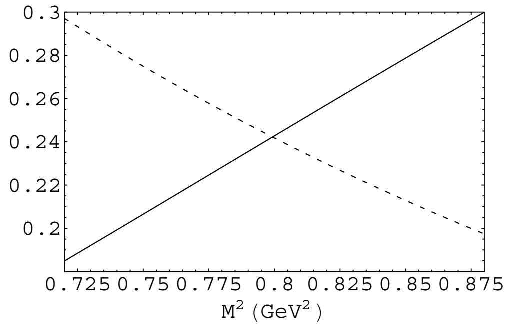
4.2 Determination of the second moment of $\phi_{p,\sigma}^\pi$ and the odd moment of $\phi_{p,\sigma}^K$

Let us consider the second moments of ϕ_p^π and ϕ_σ^π for the pion. Just as the determination of the normalization constants in the above paragraphs, one should find a window for each moment in the corresponding sum rule. The Borel windows in Table 2 are obtained under the requirement that both the contributions from continuous states and the dimension-six condensate are less than 30%. As an example, we plot the results for the moment $\langle \xi_p^2 \rangle$ in Figs. 2a,b and the numerical results are listed in Table 2.

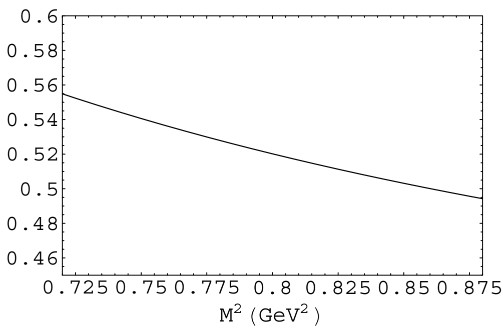
Now we turn to the determination of the first moments of $\phi_{p,\sigma}^K$. The contributions from the dimension-6 condensate and the continuous states of $\langle \zeta_p^1 \rangle$ and $\langle \zeta_\sigma^1 \rangle$ are plotted in Fig. 3. For $\langle \zeta_p^1 \rangle$, the dimension-six contribution is less

Table 2. Second moments of $\phi^{\pi}_{p,\sigma}$, odd moments of $\phi^{K}_{p,\sigma}$ and their corresponding Borel windows

	$\langle\xi_p^2\rangle$	$\langle\xi_\sigma^2\rangle$	$\langle\zeta_p^1\rangle$	$\langle\zeta_\sigma^1\rangle$
	0.52 ± 0.03	0.34 ± 0.03	-0.10 ± 0.03	-0.13 ± 0.04
M^2 (GeV^2)	0.72–0.88	0.71–0.84	0.80–1.85	0.77–1.53

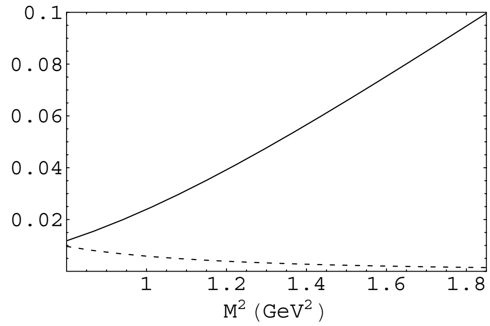


a

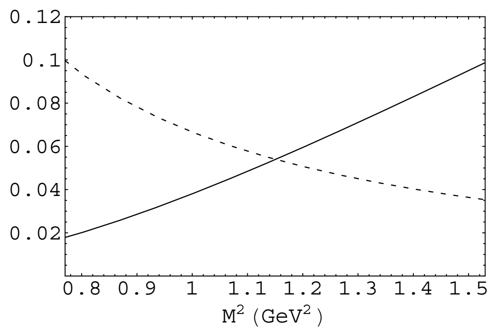


b

Fig. 2. **a** the window for the moment $\langle\xi_p^2\rangle$, the dashed and the solid line indicate the ratio of the contributions of dimension-6 condensates and continuous states in the total sum rule respectively; **b** the moment $\langle\xi_p^2\rangle$ within the Borel window



a $\langle\zeta_p^1\rangle$



b $\langle\zeta_\sigma^1\rangle$

Fig. 3. The windows for $\langle\zeta_p^1\rangle$ and $\langle\zeta_\sigma^1\rangle$. The dashed and the solid lines indicate the ratios of the contributions of dimension-6 condensates and the continuous states in the corresponding total sum rule respectively

than 1% and the continuum contribution is less than 10%. For $\langle\zeta_\sigma^1\rangle$, the contributions of dimension-six condensate and continuous states are less than 10%.

With these windows we can get the values of the corresponding moments. These results are listed in Table 2.

4.3 Determination of the fourth moment of $\phi^{\pi}_{p,\sigma}$ and the second moment of $\phi^{K}_{p,\sigma}$

Now we consider the second moment $\langle\zeta_p^2\rangle$ of ϕ_p^K for the K meson. The Borel window for $\langle\zeta_p^2\rangle$ is shown in Fig. 4a when the contributions of continuous states and the dimension-six condensate are less than 30%. The numerical results are listed in Table 3.

However, for the fourth ($n=2$) moments of $\phi_p^\pi, \phi_\sigma^\pi$ of the π meson, we cannot find the Borel windows when the contributions of continuous states and the dimension-six condensate are required to be less than 30%. For the second ($n=1$) moment of ϕ_σ^K of the K meson, we find that the Borel window is very narrow when the contributions of continuous states and the dimension-six condensate are required to be less than 30%. As we relax the requirement that the contributions of continuous states and the dimension-six condensate are less than 35%, the Borel windows for $\langle\zeta_\sigma^2\rangle$ and $\langle\xi_p^4\rangle$ can be found. For $\langle\xi_\sigma^4\rangle$, one can find the Borel window only when the contributions of continuous states and the dimension-six condensate are less than 40%. The above windows are shown in Figs. 4b–d.

The values of these moments within their corresponding windows are listed in Table 3.

4.4 From 2-particle distribution amplitudes to 3-particle distribution amplitude

There are three twist-3 distribution amplitudes ϕ_p^π, ϕ_σ^π and $\phi_{3\pi}$ for the π meson. As shown in [4], they are not inde-

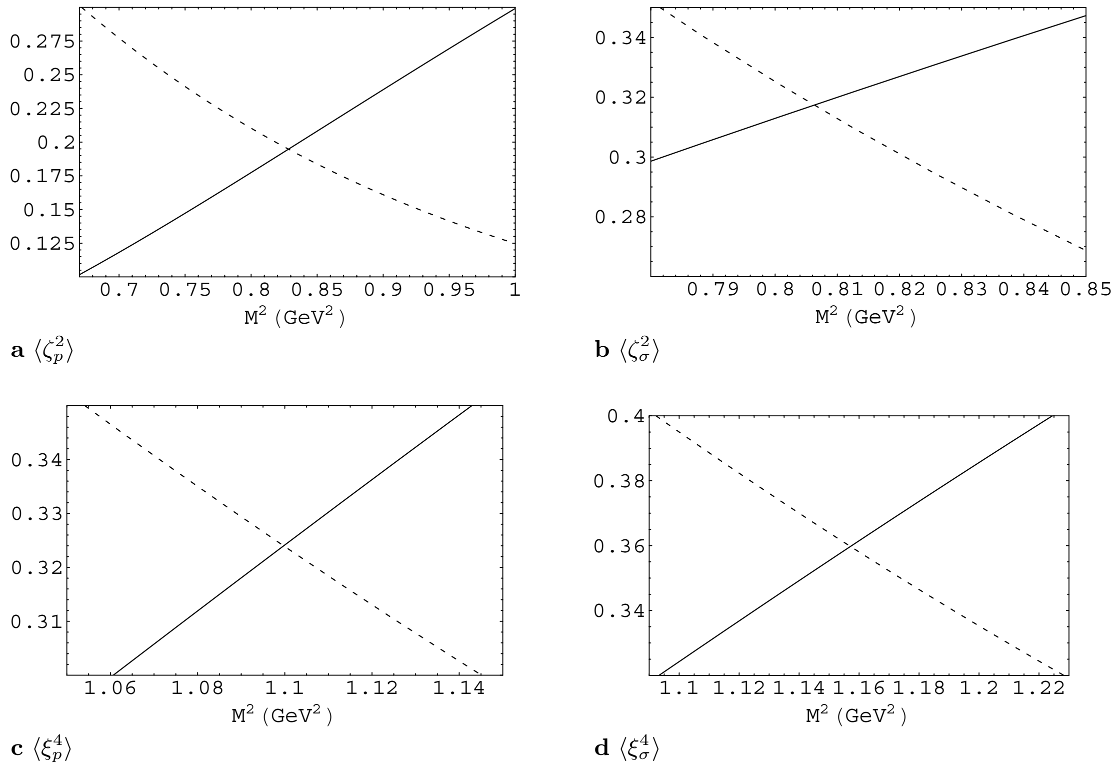


Fig. 4. The windows for the second moments of ϕ_p^K, ϕ_σ^K and the fourth moments of $\phi_p^\pi, \phi_\sigma^\pi$. The dashed and the solid lines indicate the ratios of the contributions of dimension-6 condensates and the continuous states in the corresponding total sum rule respectively

Table 3. Fourth moments of $\phi^\pi_{p,\sigma}$ and second moments of $\phi^K_{p,\sigma}$. But note that the values of $\langle\xi_p^4\rangle$ and $\langle\zeta_\sigma^2\rangle$ given in this table are under the requirement of 35% uncertainty and $\langle\xi_\sigma^4\rangle$ is under the requirement of 40% uncertainty

	$\langle\xi_p^4\rangle$	$\langle\xi_\sigma^4\rangle$	$\langle\zeta_p^2\rangle$	$\langle\zeta_\sigma^2\rangle$
	0.44 ± 0.01	0.20 ± 0.01	0.43 ± 0.04	0.173 ± 0.002
M^2 (GeV^2)	1.06–1.14	1.08–1.22	0.67–1.00	0.78–0.85

pendent. By employing the equations of motion in QCD, one can obtain some relations between them. For the pion, the relations between the twist-3 distribution amplitudes of the lowest Fock state and the 3-particle one are given in [4]. They obtained two distribution amplitudes of the lowest Fock state $\phi^\pi_{p,\sigma}$ from the 3-particle distribution amplitude $\phi_{3\pi}$ which was given by a direct calculation in the QCD sum rule method [16]. On the contrary, we use the relations from the equations of motion to see what we can say about the 3-particle distribution amplitudes with the above results of the distribution amplitudes of the lowest Fock state as input. The results can also be compared with those obtained by the QCD sum rule directly [16]. The cross checks in these calculations are helpful to judge upon the accuracy of the sum rules.

First, let us discuss the pion case. The 3-particle distribution amplitude of the π meson can be defined as

$$\langle 0|\bar{d}(x)\sigma_{\mu\nu}\gamma_5 g G_{\alpha\beta}(-vx)u(-x)|\pi^+(q)\rangle$$

$$= \mathrm{i} f_{3\pi}\left[q_\alpha(q_\mu\delta_{\nu\beta} - q_\nu\delta_{\mu\beta}) - (\alpha \leftrightarrow \beta)\right] \times \int \mathcal{D}\alpha_i\, \mathrm{e}^{\mathrm{i}qx(-\alpha_1+\alpha_2+v\alpha_3)}\phi_{3\pi}(\alpha_i), \qquad (29)$$

where $\mathcal{D}\alpha_i = \mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\delta(\alpha_1+\alpha_2+\alpha_3-1)$. There is the system of recurrence relations for the moments $\langle\xi_p^n\rangle$ and $\langle\xi_\sigma^n\rangle$ [4]:

$$\langle\xi_p^n\rangle = \delta_{n0} + \frac{n-1}{n+1}\langle\xi_p^{n-2}\rangle + 2R_p(n-1)\int_{-1}^1 \mathrm{d}v \langle\!\langle(\alpha_2-\alpha_1+v\alpha_3)^{n-2}\rangle\!\rangle - 2R_p\frac{(n-1)(n-2)}{n+1} \times \int_{-1}^1 \mathrm{d}v\, v\langle\!\langle(\alpha_2-\alpha_1+v\alpha_3)^{n-3}\rangle\!\rangle, \qquad (30)$$

$$\langle\xi_\sigma^n\rangle = \delta_{n0} + \frac{n-1}{n+3}\langle\xi_\sigma^{n-2}\rangle \qquad (31)$$

$$+ 6R_\sigma \frac{n-1}{n+3}\int_{-1}^{1} \mathrm{d}v \langle\!\langle(\alpha_2-\alpha_1+v\alpha_3)^{n-2}\rangle\!\rangle$$
$$- 6R_\sigma \frac{n}{n+3}\int_{-1}^{1} \mathrm{d}v\, v \langle\!\langle(\alpha_2-\alpha_1+v\alpha_3)^{n-1}\rangle\!\rangle,$$

where $\langle\!\langle(\alpha_2-\alpha_1+v\alpha_3)^n\rangle\!\rangle = \int \mathcal{D}\alpha_i\, \phi_{3\pi}(\alpha_i)\,(\alpha_2-\alpha_1+v\alpha_3)^n$ defines the moments of the 3-particle distribution amplitude. Instead of taking $R_p = R_\sigma = R$ as in [4], we introduce them separately,

$$R_p = \frac{1}{m^p_{0\pi}}\frac{f_{3\pi}}{f_\pi} \quad \text{and} \quad R_\sigma = \frac{1}{m^\sigma_{0\pi}}\frac{f_{3\pi}}{f_\pi}.$$

Now, we take second moments into account. The above relation can be reduced to

$$\langle\xi^2_\sigma\rangle = \frac{1}{5}\langle\xi^0_\sigma\rangle + \frac{12}{5}R_\sigma - \frac{8}{5}R_\sigma\langle\!\langle\alpha_3\rangle\!\rangle$$

and

$$\langle\xi^2_p\rangle = \frac{1}{3}\langle\xi^0_p\rangle + 4R_p,$$

which gives, from Tables 1 and 2,

$$\langle\!\langle\alpha_3\rangle\!\rangle = (0.13,\, 0.27)\,, \quad f_{3\pi} = (0.0049,\, 0.0067)\,\mathrm{GeV}^2. \tag{32}$$

At this point, we compare the moment $\langle\!\langle\alpha_3\rangle\!\rangle$ and $f_{3\pi}$ with those calculated directly by the sum rule method in [16]: $\langle\!\langle\alpha_3\rangle\!\rangle = (0.06, 0.22)$, $f_{3\pi} \approx 0.0035\,\mathrm{GeV}^2$. One can see that the results from the two approaches are compatible with each other to the order of magnitude.

From the analysis in previous section, we have shown that the fourth moments $\langle\xi^4_p\rangle$ and $\langle\xi^4_\sigma\rangle$ cannot be obtained in a reliable way, so we do not use them to give the other moments, i.e., $\langle\!\langle\alpha_1^2\rangle\!\rangle$ and $\langle\!\langle\alpha_1\alpha_2\rangle\!\rangle$, etc.

Now we turn to the K meson case. Similar to the pionic case, one can define a 3-particle distribution amplitude:

$$\begin{aligned}&\langle 0|\bar{s}(x)\sigma_{\mu\nu}\gamma_5 g G_{\alpha\beta}(-vx)u(-x)|K^+(q)\rangle\\ &= \mathrm{i} f_{3K}[q_\alpha(q_\mu\delta_{\nu\beta}-q_\nu\delta_{\mu\beta})-(\alpha\leftrightarrow\beta)]\\ &\times \int \mathcal{D}\alpha_i\, \mathrm{e}^{\mathrm{i}qx(-\alpha_1+\alpha_2+v\alpha_3)}\phi_{3K}(\alpha_i).\end{aligned} \tag{33}$$

Following [4], a similar recurrence relation can be obtained. As the first and second moments of $\phi^K_{p,\sigma}$ are taken into account, the recurrence relation can be truncated to three equations,

$$\langle\zeta^1_\sigma\rangle = \frac{3}{4}\frac{R'_\sigma}{R'_p}\langle\zeta^1_p\rangle\,, \tag{34}$$

$$\langle\zeta^2_\sigma\rangle = \frac{3}{5}\frac{R'_p}{R'_\sigma}\langle\zeta^2_p\rangle - \frac{8}{15}R'_p\langle\!\langle\alpha_3\rangle\!\rangle_K, \tag{35}$$

$$\langle\zeta^2_p\rangle = \frac{1}{3}\frac{R'_p}{R'_\sigma}\langle\zeta^0_\sigma\rangle + 4R'_p, \tag{36}$$

where $R'_{p,\sigma} = f_{3K}/(f_K m^{p,\sigma}_{0K})$, and the primes on the R and the subscript K indicate that the quantities are related to the K meson. From Table 1, we have $R'_\sigma/R'_p \approx 1.06/0.71$, so (34) is a direct constraint of the two first moments. Our calculation (see Table 2) shows that the left hand side of (34) is about -0.13 and the right hand side is about -0.11. It can be seen that this equation is approximately fufilled. Solving the last two equations, (35) and (36), we can obtain f_{3K} and $\langle\!\langle\alpha_3\rangle\!\rangle_K$:

$$\begin{aligned} f_{3K} &= (0.0071,\, 0.0105)\,\mathrm{GeV}^2,\\ \langle\!\langle\alpha_3\rangle\!\rangle_K &= (5.01,\, 5.37).\end{aligned} \tag{37}$$

To determine more moments of the 3-particle distribution amplitude, we have to include higher moments of the 2-particle distribution amplitudes. However, one cannot guarantee the convergence of the operator expansion for bigger n.

5 Summary and discussion

In this paper we calculate the first three moments of the twist-3 distribution amplitudes $\phi^\pi_{p,\sigma}$ and $\phi^K_{p,\sigma}$ by using the QCD sum rules. It has been shown that the first three moments of ϕ^K_p and the first two moments of ϕ^π_p and $\phi^{\pi,K}_\sigma$ of the pion and kaon can be obtained with 30% uncertainty. The fourth moments $\langle\xi^4_{p,\sigma}\rangle$ of $\phi^\pi_{p,\sigma}$ and the second moment $\langle\zeta^2_\sigma\rangle$ of ϕ^K_σ can be obtained under 35%–40% uncertainty. When the α_s-corrections (we take them from [9]) to the perturbative part of m^p_{0K}, $m^p_{0\pi}$ are included, we find that the values of $m^p_{0\pi}$ and m^p_{0K} are increased (and the corresponding Borel windows become a little narrower) to $m^p_{0K} = 1.25 \pm 0.15\,\mathrm{GeV}$ and $m^p_{0\pi} = 1.10 \pm 0.08\,\mathrm{GeV}$. It may be expected that the α_s-corrections to the perturbative parts in the sum rules for m^σ_{0K} and $m^\sigma_{0\pi}$ will be about 15–20%.

As to the normalization constants $m^{p,\sigma}_{0\pi}$ and $m^{p,\sigma}_{0K}$, our calculated results show that they are smaller than the values which are given by the equations of motion and at the same time, the calculated $m^\sigma_{0\pi,K}$ are smaller than the corresponding $m^p_{0\pi,K}$. These deviations can be traced to the non-perturbative condensate effects (see the sum rules (16), (17), (25) and (27) for the normalization constants), in particular, the dimension-six condensate terms in opposite sign lead to about 30% difference between these normalization constants. On the other hand, from the sum rules, one can see that the contributions from the continuous state grow too fast, which prevents us from taking M^2 to be larger values (larger M^2 will lead to bigger values of the normalization constants), and then the window for the sigma sum rules ($m^\sigma_{0\pi,K}$) are much narrower than the non-sigma sum rules ($m^p_{0\pi,K}$). So we think the smaller values of $m^\sigma_{0\pi,K}$ may be related to our approximation in the hadronic spectrum representation.

Furthermore, we calculate the moments of the quark–antiquark–gluon distribution amplitude from the numerical results on the distribution amplitudes of the lowest Fock state by applying the exact equations of motion and compare our results with those from [16]. The comparison shows that they are compatible with each other to the order of magnitude. It is helpful to improve the accuracy of the QCD sum rule approach for getting more precise information on the twist-3 distribution amplitudes.

These moments can provide several constraints upon the twist-3 distribution amplitudes. These constraints will be helpful for building the model of the distribution amplitude. For example, [17] suggests a model for the twist-3 wave function of the pion based on the QCD sum rule calculation to get a more realistic contribution to the pion form factor. Here we discuss the distribution amplitude ϕ_p^K of the kaon (since the first three moments can be obtained reliably). As usual, we expand the distribution amplitudes in Gegenbauer's polynomials and use the moments to determine their first few coefficients in a truncated form: $\phi_p^K(\zeta) = \sum_{n=0}^{2} C_n^{1/2}(\zeta)a_n$. From the three moments of ϕ_p^K (see Tables 2 and 3), we have the twist-3 distribution amplitude approximately,

$$\phi_p^K(\zeta) = 1 - 0.30\ C_1^{1/2}(\zeta) + 0.73\ C_2^{1/2}(\zeta), \qquad (38)$$

which is asymmetric since the broken effects of $SU(3)_f$ symmetry are taken into account.

Acknowledgements. We would like to thank A. Khodjamirian for his helpful communication. This work was supported in part by the National Science Foundation of China, 10275070.

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